4 The new Keynesian model

4.2 The New keynesian model

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The New keynesian model

- In the New Keynesian model we will again have an infinitely lived representative household
- There will be a basket of differentiated goods and monopolistic competition - producers set prices
- The model incorporates nominal rigidities - only a fraction of firms can re-set prices in each period (Calvo Pricing)

A first distortion in the model is the existence of **monopolistic competition** - differentiated goods:

$$MaxE_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{2}$$

$$\tag{3}$$

where $\epsilon > 0$ is the elasticity of substitution among goods. The budget constraint is given by:

$$\int_0^1 P_t(i)C_t(i)di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \tag{4}$$

where T_t corresponds to income such as dividends, net of taxes.

In spite of the additional complication, it can be shown that optimization implies(annex 3.1 in Gali):

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \tag{5}$$

And that for optimising households:

$$\int_0^1 P_t(i)C_t(i)di = P_t C_t \tag{6}$$

So that the budget constraint can be written as before as:

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \tag{7}$$

Maximization delivers the usual conditions:

$$MaxE_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{8}$$

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \tag{9}$$

$$Q_t = \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right) \tag{10}$$

Let us again assume again the utility function:

$$U(C_t, N_t) = \frac{C_t^{(1-\sigma)}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{11}$$

Then we have the same conditions as in the monetary model:

$$c_t \simeq E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho) \tag{12}$$

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{13}$$

The New keynesian model - firms

Firms face three constraints:

- The production function
- A downward sloping demand curve

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \tag{14}$$

- Staggered price setting (Calvo pricing)

The New keynesian model - firms

Now we have a continuum of firms each producing:

$$Y_t(i) = A_t N(i)^{1-\alpha} \tag{15}$$

where A_t is the level of technology. $a_t = logA_t$ is an exogenous shock process. The main new assumption is that of Calvo price rigidity: only a fraction $(1-\theta)$ of firms change prices in each period:

$$\Pi_t^{1-\epsilon} = \theta * 1 + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \tag{16}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is gross inflation and P_t^* is the price chosen by optimising firms A log-linear approximation around the zero inflation steady state implies:

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \tag{17}$$

The problem of the firms

Optimization is more difficult now because firms have to choose P_t^* taking into account the probability of **not** being able to change prices in the future and the evolution of future costs:

$$Max \sum_{k=0}^{\infty} \theta^k E_t \left(Q_{t,t+k}(P_t^* Y_{t+k|t} - \Psi_t(Y_{t+k|t})) \right) \tag{18}$$

Note that θ is the probability of the firm not being able to reset prices in each period. The firm has no way of influencing this probability. Note also that this problem is the same for all firms so reoptmising firms will all chose the same price P_t^* (thus i was dropped above). $\Psi_t(Y_{t+k|t})$ is the cost function.

The problem of the firms

The future demand for the firm's goods will depend on the relation of the price it has chosen in period t (P_t^*) and the average price of goods in the economy at each moment in time, taking into account good differentiation:

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \tag{19}$$

The New keynesian model - optimization

Note that:

$$P_t^* Y_{t+k|t} = \left(\frac{P_t^{*\,1-\epsilon}}{P_{t+k}^{-\epsilon}} \right) C_{t+k} \tag{20}$$

The derivative of $P_t^* Y_{t+k|t}$ relative to P_t^* is:

$$(1-\epsilon) \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} = (1-\epsilon) Y_{t+k|t} \tag{21}$$

and the derivative of $Y_{t+k|t}$ relative to P_t^* is:

$$-\epsilon \left(\frac{P_t^{*\,-\epsilon-1}}{P_{t+k}^{-\epsilon}} \right) C_{t+k} = -\epsilon Y_{t+k|t} P_t^{*\,-1} \tag{22}$$

Going back to:

$$Max \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k}(P_t^* Y_{t+k|t} - \Psi_t(Y_{t+k|t})) \right\} \tag{23}$$

The first order condition is:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k}((1-\epsilon) Y_{t+k|t} - \psi_{t+k|t}(-\epsilon Y_{t+k|t} P_t^{-1})) \right\} = 0 \tag{24}$$

where $\psi_{t+k|t}$ is the nominal marginal cost. This simplifies to:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t}((1-\epsilon) - \psi_{t+k|t}(-\epsilon P_t^{*-1})) \right\} = 0 \tag{25}$$

Multiply everything by P_t^* and divide by $(1-\epsilon)$:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} (P_t^* - \psi_{t+k|t} \frac{-\epsilon}{1-\epsilon}) \right\} = 0 \tag{26}$$

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} (P_t^* - \psi_{t+k|t} \frac{\epsilon}{\epsilon-1}) \right\} = 0 \tag{27}$$

$$\sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k} Y_{t+k|t} (P_t^* - \mathcal{M} \psi_{t+k|t}) \} = 0 \tag{28}$$

where $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$

No price rigidities ($\theta = 0$)implies:

$$P_t^* = \mathcal{M}\psi_{t|t} \tag{29}$$

here note that monopolistic competition introduces a markup $\mathcal{M}$ relative to marginal cost. Note that this is the frictionless markup, but still is a distortion.

$\Psi_t(Y_{t+k|t}$ is the cost function. Note that optimizing firms firms will choose labour N in each period optimally (i.e. minimizing costs) given the level of production. The firms will solve the following problem (in real terms):

$$Min \left(\frac{W_t}{P_t} \right) N_t + MC_t(Y_t - A_t N_t^{1-\alpha}) \tag{30}$$

Thus real marginal cost will be:

$$\frac{W_t}{P_t} - MC_t((1-\alpha)A_t N_t^{-\alpha}) = 0 \tag{31}$$

$$MC_t = \frac{W_t}{P_t} / \left((1-\alpha)A_t N_t^{-\alpha}\right) \tag{32}$$

Monetary policy - linearise around steady state

Divide:

$$\sum_{k=0}^{\infty} \theta^k E_t \left(Q_{t,t+k} Y_{t+k|t} (P_t^* - \mathcal{M} \psi_{t+k|t}) \right) = 0 \tag{33}$$

by P_{t-1} and letting $\Pi_{t,t+k} = \frac{P_t+k}{P_t}$

$$\sum_{k=0}^{\infty} \theta^k E_t \left(Q_{t,t+k} Y_{t+k|t} \left(\frac{P_t^*}{P_{t-1}} - \mathcal{M} MC_{t+k|k} \Pi_{t-1,t+k} \right) \right) = 0 \tag{34}$$

Monetary policy - steady state

In steady state, assuming zero inflation:

- $\frac{P_t^*}{P_{t-1}} = 1$
- $\Pi_{t-1,t+k} = 1$
- $P_t^* = P_{t+k}$
- $Y_{t+k|t} = Y$
- $MC_{t+k|t} = MC$
- $Q_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\sigma} \frac{P_t}{P_{t+k}} = \beta^k$

Log-linearization of price setting equation

Write:

$$\sum_{k=0}^{\infty} \theta^k E_t(Q_{t,t+k} Y_{t+k|t}(\frac{P_t^*}{P_{t-1}} - \mathcal{M} MC_{t+k|t} \Pi_{t-1,t+k})) = 0 \qquad (35)$$

Note that in steady state: $\frac{P_t^*}{P_{t-1}} = 1$ and $\Pi_{t-1,t+k} = 1$ and so: $\mathcal{M} MC = 1 \Leftrightarrow MC = 1/\mathcal{M}$

Firms price setting

The log-linearization around a zero inflation steady state leads to the following equation:

$$p_t^* - p_{t-1} = \sum_{k=0}^{\infty} (\theta\beta)^k (\hat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})) \qquad (36)$$

where $\hat{mc}_{t+k|t} = mc_{t+k|t} - mc$ and $mc = -\mu$, $\mu = log\mathcal{M}$ we can also write this equation as:

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\theta\beta)^k E_t(\hat{mc}_{t+k|t} + p_{t+k}) \qquad (37)$$

Firms price setting

It is possible to show that:

$$p_t^* - p_{t-1} = \sum_{k=0}^{\infty} (\theta\beta)^k (\hat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})) \tag{38}$$

in combination with (17), inflation can be approximated as:

$$\pi_t = \beta_t E_t \pi_{t+1} + \lambda \hat{mc}_t \tag{39}$$

$$\lambda = \frac{[(1-\theta)(1-\beta\theta)]}{\theta}\Theta \tag{40}$$

$$\Theta = \frac{(1-\alpha)}{1-\alpha+\alpha\epsilon} \tag{41}$$

Equilibrium

Market clearing implies:

$$Y_t = C_t \tag{42}$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho) \tag{43}$$

$$y_t = a_t + (1-\alpha) n_t \tag{44}$$

Equilibrium

From:

$$MC_t = \frac{W_t}{P_t}/((1-\alpha)A_t N_t^{-\alpha}) \tag{45}$$

taking logs:

$$mc_t = w_t - p_t - (a_t - \alpha n_t) - log(1-\alpha) \tag{46}$$

Real marginal cost

Noting that $y_t - n_t = a_t - \alpha n_t$ we can write

$$mc_t = (\sigma y_t + \varphi n_t) - (y - \alpha n_t) - log(1 - \alpha) \quad (47)$$

$$mc_t = (\sigma + \frac{1+\varphi}{1-\alpha})y_t - \frac{1+\varphi}{1-\alpha}a_t - log(1 - \alpha) \quad (48)$$

Equilibrium

Now define the marginal cost under flexible prices $(= -\mu, \mu = log\mathcal{M})$ as:

$$mc = (\sigma + \frac{1+\varphi}{1-\alpha})y_t^n - \frac{1+\varphi}{1-\alpha}a_t - log(1-\alpha) \quad (49)$$

where y_t^n is the natural rate of output. Then the marginal cost in deviations from steady state can be written as:

$$\hat{mc}_t = (\sigma + \frac{1+\varphi}{1-\alpha})(y_t - y_t^n) \quad (50)$$

Where $\tilde{y}_t = (y_t - y_t^n)$ is the output gap.

Recalling that:

$$\pi_t = \beta_t E_t \pi_{t+1} + \lambda \hat{mc}_t \tag{51}$$

One can write (New Keynesian Phillips Curve):

$$\boxed{\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t} \tag{52}$$

where $\kappa = \lambda(\sigma + \frac{\varphi+\alpha}{1-\alpha})$

The **Dynamic IS equation** is given by:

$$\boxed{\tilde{y}_t = E_t \tilde{y_{t+1}} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^n)} \tag{53}$$

where :

$$r_t^n = \rho + \sigma E_t \Delta y_{t+n}^n \tag{54}$$

Solving the equation forward:

$$\tilde{y}_t = -\frac{1}{\sigma} \sum_{k=0}^{\infty} (r_{t+k} - r_{t+k}^n) \tag{55}$$

Summary

The Essential elements of the New keynesian model can be summarised in three equations:

- **New Keynesian Phillips Curve**:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t \quad (56)$$

  where $\kappa = \lambda(\sigma + \frac{\varphi+\alpha}{1-\alpha})$

- The **Dynamic IS equation** is given by:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^n) \quad (57)$$

- **Monetary policy rule**:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \upsilon_t \quad (58)$$

  where υ_t is a monetary policy shock.